

B.Sc. Semester - 1 (NEP-2020) Examination
OCT/NOV-2025 (As Per Syllabus Year June-2025)
MATH: Calculus and Solid Geometry (Major-2)

Time: 2:00 Hours

Marks:50

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Que.1(A) Answer any one (05)

- (1) State and prove Lagrange's mean value theorem with Geometrical Interpretation.
- (2) Verify Cauchy's mean value theorem for $f(x) = (x-1)^2$ and $g(x) = x(x-1)^3, \forall x \in [0,2]$ also find value of c if exist.

Que.1(B) Answer any one (05)

- (1) State Taylor's theorem and derive power series expansion of $f(x) = \sin x, x \in R$
- (2) Show that $e^x \sin x = x + \frac{x^2}{1} + \frac{x^3}{3} - \frac{x^5}{30} + \dots + (\sqrt{2})^n \sin \frac{n\pi}{4} \cdot \frac{x^n}{n!} + \dots$

Que.2(A) Answer any one (05)

- (1) State L'Hospital's rule for $\frac{\infty}{\infty}$ form and evaluate $\lim_{x \rightarrow 0} \frac{\log \sin 2x}{\log \sin x}; (\sin x, \sin 2x > 0)$
- (2) Evaluate $\lim_{x \rightarrow 0} \left(\cot^2 x - \frac{1}{x^2} \right)$

Que.2(B) Answer any one (05)

- (1) If $\lim_{x \rightarrow \frac{\pi}{2}} \frac{\cos^2 x}{a - b \csc x} = 1$ then find value of a and b.
- (2) Evaluate $\lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right)^{\frac{6}{x^2}}$

Que.3(A) Answer any one (05)

- (1) Find the equation of the sphere passing through the point $O(0,0,0), A(-a, b, c), B(a, -b, c)$ and $C(a, b - c)$.
- (2) Show that two circles $x^2 + y^2 + z^2 - y + 2z = 0; x - y + z - 2 = 0$ and $x^2 + y^2 + z^2 + x - 3y + z - 5 = 0; 2x - y + 4z - 1 = 0$ lie on the same sphere, also find its equation.

Que.3(B) Answer any one (05)

- (1) Find the co-ordinates of the point where the line $\frac{x-1}{2} = \frac{y-6}{3} = \frac{z-4}{4}$ intersects the sphere $x^2 + y^2 + z^2 = 10$
- (2) Show that $2x - 2y + z + 12 = 0$ touches the sphere $x^2 + y^2 + z^2 - 2x - 4y + 2z - 3 = 0$ and find the point of contact.

Que.4(A) Answer any one (05)

- (1) Derive equation of a cylinder of which generator remain parallel to $\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$ and passing through $ax^2 + 2hxy + by^2 + 2gz + 2fy + c = 0; z = 0$.
- (2) Find equation of cylinder whose generator is parallel to $\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$ and enveloping curve is $x^2 + y^2 + z^2 = a^2$.

Que.4(B) Answer any one (05)

- (1) Find equation of cylinder whose generator is parallel to $X - axis$ and touching the sphere having centre $(1,2,0)$ and radius 3.
- (2) Find equation of cylinder whose generator is parallel to $\frac{x}{3} = \frac{y}{1} = \frac{z}{-2}$ and passing through $y^2 + 2z^2 = 1; x = 3$.

Que.5(A) Answer any one (05)

- (1) For $x > 0$ show that $\frac{x}{1+x^2} < \tan^{-1}x < x$.
- (2) Assuming the validity of expansion, Expand $\log x$ in powers of $(x - 1)$.

Que.5(B) Answer any one (05)

- (1) Find Co-ordinates of the points where the line $2x - 1 = y + 3 = 4 - z$ intersects the sphere $x^2 + y^2 + z^2 - 6x + 8y - 4z + 4 = 0$.
- (2) Find the enveloping cylinder of the sphere $x^2 + y^2 + z^2 - 2x + 4y - 1 = 0$ having its generator parallel to the straight line $x = y = z$.
